

ETERNAL EXPANSION OF CLOSED UNIVERSE

O. B. Karpov

*Moscow State Mining University,
119991, Moscow, Russia*

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The potential barrier of the closed universe expansion has been investigated and its overcoming condition has been obtained. The restrictions on the Friedmann integrals, cosmological constant and medium components energy densities have been analyzed. The phase-space has been considered and the phase curves of eternally expanding closed universes have been plotted. A questionable coincidence of the our Universe Friedmann integrals has been discussed.

1 Introduction

As it is known, data on distance modulus versus redshift reveal the presence of an acceleration of the cosmological expansion [1,2,3]. This is possible in existence of a cosmological repulsion which is described usually by Λ -term or equivalently by the presence of an vacuum-like medium with a negative pressure. An open universe will expand forever even if $\Lambda = 0$. A closed universe at $\Lambda = 0$ will stop expanding in future and begin to recollapse without fail. If $\Lambda > 0$, the expansion becomes accelerated when the doubled vacuum density exceeds the decreasing matter density. The open universe is sure to pass this condition and its eternal expansion has not an alternative¹. Reaching of the closed universe the acceleration state and its consequent eternal expansion requires a potential barrier overcoming which is possible under certain conditions. In this work a restriction on the closed universe parameters being necessary to getting over the barrier and the universe dynamics in this case are investigated.

¹In the work [4] the possibilities of recollapse of a flat universe owing to decay of cosmological constant, collision with a null singularity and formation of space-like curvature singularity during expansion have been investigated. In this work these possibilities are not considered.

2 Condition of eternal expansion

The closed universe dynamics is described by the Einstein field equation for curvature radius a of 3-space in comoving with cosmological expansion nonrotating (synchronous) frame:

$$\dot{a}^2 = \frac{A_m}{a} + \frac{a^2}{A_v^2} - 1. \quad (1)$$

Here and below convention $c = 1$ is used. The evolution constants A_m and A_v following [5, 6] are named the Friedmann integrals. The matter integral

$$A_m = \frac{8\pi}{3}G\rho_m a^3, \quad (2)$$

where ρ_m is the dust-like matter density including a dark matter, G is the gravitational constant. The vacuum integral A_v

$$A_v^{-2} = \frac{8\pi}{3}G\rho_v = \frac{\Lambda}{3}, \quad (3)$$

where ρ_v is the vacuum energy density, Λ is the cosmological constant. The equation (1) has been written without taking into account a radiation which contribution dominates at an early stage of the universe evolution and is negligible at the

stage considered. Define

$$U_m = -\frac{A_m}{a}, \quad U_v = -\frac{a^2}{A_v^2}, \quad U = U_m + U_v$$

and write the equation (1) in form of an "energy conservation law"

$$\dot{a}^2 + U(a) = -1.$$

Graphs of the functions $U_m(a)$, $U_v(a)$, $U(a)$ are plotted in Fig. 1. At the point $a = \tilde{a}$,

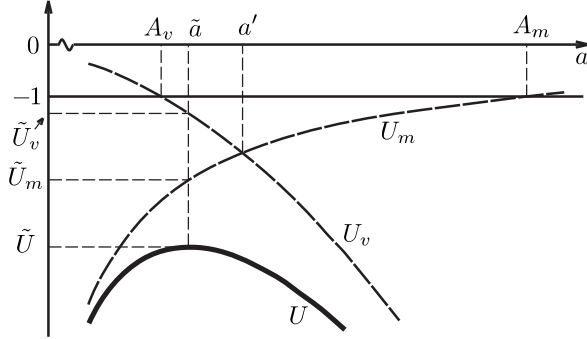


Figure 1: Potential barrier

$$\tilde{a}^3 = \frac{1}{2} A_m A_v^2 \quad (4)$$

the maximum \tilde{U} of the potential barrier $U(a)$ is disposed,

$$\tilde{U} = U(\tilde{a}) = -\left(\frac{\alpha A_m}{A_v}\right)^{2/3}, \quad (5)$$

where $\alpha = 3\sqrt{3}/2$. At this point the balance of matter gravitation and vacuum antigravitation is achieved, $\rho_m = 2\rho_v$. The levels \tilde{U} , \tilde{U}_m , \tilde{U}_v , 0 are equidistant:

$$\tilde{U} - \tilde{U}_m = \tilde{U}_m - \tilde{U}_v = \tilde{U}_v = \frac{1}{3}\tilde{U}.$$

At the point $a = \tilde{a}$ the deceleration parameter changes the sign:

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{\tilde{a}^3 - a^3}{a^3 - aA_v^2 + 2\tilde{a}^3}. \quad (6)$$

At $a < \tilde{a}$ the matter gravitation dominates and $q > 0$, at $a > \tilde{a}$ the vacuum antigravitation dominates and $q < 0$. At the point $a = a'$,

$$a'^3 = 2\tilde{a}^3$$

$U'_m = U'_v$ and $\rho_m = \rho_v$. If the level $U = -1 > U'_m = U'_v$ then $A_m > A_v$ (as represented in Fig. 1); $A_m \leq A_v$ otherwise. The Friedmann integral A_m is a maximum value of the curvature radius in the standard model $\Lambda = 0$, the integral A_v equals an initial value a in the de Sitter universe $\rho_m = 0$:

$$a(t) = A_v \cosh(t/A_v). \quad (7)$$

A closed universe will expand eternally if the level $U = -1$ is above the potential barrier summit $U = \tilde{U}$ (5), $|\tilde{U}| \geq 1$ (the equality sign corresponds to the asymptotic approaching a to \tilde{a} (4)):

$$\frac{\alpha A_m}{A_v} \geq 1. \quad (8)$$

The condition of potential barrier overcoming corresponds to the strict inequality. Thus, universe fate is determined by the Friedmann integrals ratio. The condition (8) means the limitation on the cosmological constant

$$\Lambda \geq \frac{4}{3} A_m^{-2}. \quad (8')$$

The A_m value (2) can be estimated very rough due to a considerable uncertainty of the scale factor a . According to [5,6] $A_v \sim A_m$ and even some more, and then a fulfilment of the condition (8) turns out to be problematic.

Find out the condition (8) for the density parameters Ω_m and Ω_v ,

$$\Omega_m = \frac{\rho_m}{\rho_c} = \left(1 + \left(\frac{a}{a'}\right)^3 - \frac{a}{a'} \left(\frac{A_v}{A_m}\right)^{\frac{2}{3}}\right)^{-1} \quad (9)$$

$$\Omega_v = \frac{\rho_v}{\rho_c} = \left(\frac{a}{a'}\right)^3 \Omega_m = (HA_v)^{-2}, \quad (10)$$

where ρ_c is the critical density, $\rho_c = 3H^2/8\pi G$, $H = \dot{a}/a$ is the Hubble constant. The density parameter Ω_m achieves a maximum at $a = a_1$

$$a_1 = \frac{A_v}{\sqrt{3}};$$

$\Omega_m = 1$ at $a = 0$ and $a = A_v$. The parameter Ω_v achieves a maximum (and the Hubble constant H - a minimum) at $a = a_2$

$$a_2 = \frac{3}{2} A_m;$$

$\Omega_v = 1$ at $a^{-1} = 0$ and $a = A_m$. The deceleration parameter (6)

$$q = \frac{1}{2}\Omega_m - \Omega_v; \quad (11)$$

$q = 0.5$ at $a = 0$ and $a = a_1$, $q = -1$ at $a^{-1} = 0$ and $a = a_2$. The curvature radii a_1 , a_2 and \tilde{a} satisfy the ratios

$$\frac{a_2}{a_1} = \left(\frac{a_2}{\tilde{a}}\right)^{3/2} = \left(\frac{\tilde{a}}{a_1}\right)^3 = \frac{\alpha A_m}{A_v}.$$

The Einstein field equation (1) connects the density parameters and the Friedmann integrals by the correlation

$$(\Omega_m + \Omega_v - 1)^3 = \left(\frac{A_v}{A_m}\right)^2 \Omega_m^2 \Omega_v. \quad (12)$$

The condition of the eternal expansion of the closed universe (8) means

$$(\Omega_m + \Omega_v - 1)^3 \leq \alpha^2 \Omega_m^2 \Omega_v. \quad (13)$$

If in accordance with the last data [7] we assume $\Omega \simeq 0.03$, where $\Omega = \Omega_m + \Omega_v$, then the equation (12) yields $A_v \sim 10^{-2} A_m$ and the condition (8), (13) is fulfilled easily.

As it is clear from (12), the ratio A_v/A_m is connected with the degree of the universe spatial flatness, and $A_v/A_m \rightarrow 0$ when $\Omega \rightarrow 1$. Therefore the problem of the cosmic coincidence of the Friedmann integrals A_m and A_v discussed in the works [5,6] seems to be not actual. Do not coincide the expansion factor a today with constant A_v either:

$$\left(\frac{a}{A_v}\right)^3 = \frac{\Omega_v}{\Omega_m} \frac{A_m}{A_v} \quad (14)$$

and $a/A_v \rightarrow \infty$ when $\Omega \rightarrow 1$. There exists the problem of coincidence of Ω_m and Ω_v that should be explained by anthropic arguments [3,8].

3 Phase curves

Curves in Fig. 2 represents the dynamics of universes according to (12). The point $(1,0)$ corresponds to a Big Bang, the point $(0,1)$ signifies a de Sitter universe. The evolution of an universe with the fixed Friedmann integrals ratio A_m/A_v

is described by a the curve in (Ω_m, Ω_v) plane. One and only one curve passes through each of the point (Ω_m, Ω_v) , excepting $(0,1)$ and $(1,0)$. It means an observational determination of the values Ω_m, Ω_v fixes the universe evolution in the model circumscribed (1), (12). The phase-space area (Ω_m, Ω_v) satisfying the condition (8), (13) is disposed between the curves 1 and 1' which correspond to the equality $A_v = \alpha A_m$. All the curves in this area corresponding the universes expansion begin at the point $(1,0)$; the evolutions are completed by the de Sitter universe $(0,1)$. On the curves 2 and 3 the correlations $A_v = 2A_m$ and $A_v = A_m$ are fulfilled respectively. Below the curve 1 universes expand from the Big Bang $(1,0)$ till a maximum value a defined left point of intersection of the straight line $U = -1$ and the curve $U(a)$ when Ω_m and Ω_v go to infinity, afterwards the universes begin to recontract and recollapse $(1,0)$. Above the curve 1' expansion goes on the other (large a) side of the potential barrier without the Big Bang from a minimum value a defined the right intersection point of the lines $U = -1$ and $U(a)$ to the de Sitter metric $(0,1)$. A passage from $(1,0)$ to $(0,1)$ on a straight line corresponds a flat universe $\Omega_m + \Omega_v = 1$. The dependence $a(t)$ (7) at $t \rightarrow \infty$ becomes exponential

$$a(t) = A_v \exp(t/A_v),$$

which describes the final fate of any expanding universe with the (Ω_m, Ω_v) values above the curve 1.

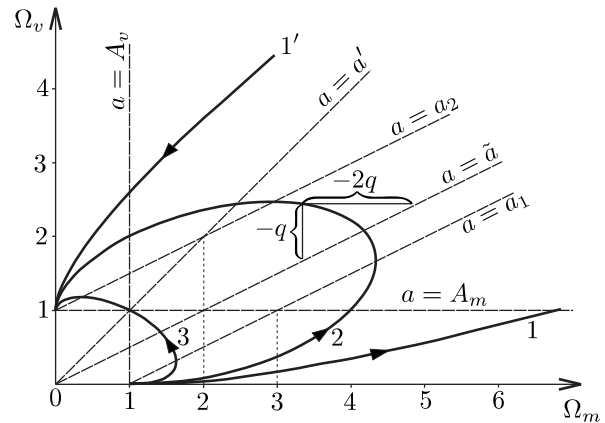


Figure 2: Phase curves

Plots of the parameters Ω_m (9), Ω_v (10), q (11) versus a/\tilde{a} for the universe $A_v = 2A_m$ (the curve 2 in Fig. 2) and for the almost flat universe $A_v = 10^{-2}A_m$ are given in Fig. 3(a) and 3(b) respectively.

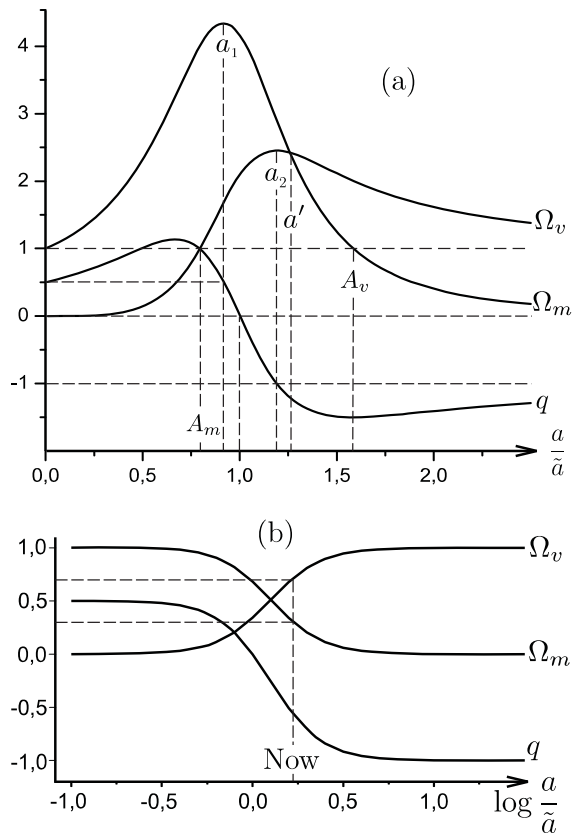
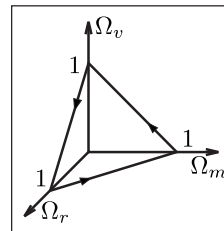


Figure 3: Universes dynamics. (a) $A_v = 2A_m$. (b) $A_v = 10^{-2}A_m$.

It should be mentioned that at an early stage of an universe evolution a radiation density ρ_r dominates, then phase-space is three-dimensional ($\Omega_m, \Omega_v, \Omega_r$). The phase curves begin at the point $(0, 0, 1)$; they are directed to the Ω_r axis for a closed universe, and when the matter grows predominant over the radiation, the curves pass on (Ω_m, Ω_v) plane as it is shown in Fig. 2. The vacuum energy density ρ_v is to prevail at a still more early stage preceding radiation-dominated, and then the universe develops from de Sitter metric to de Sitter metric along a closed phase curve. This curve simplified has been plotted here for a flat universe $\Omega_m + \Omega_v + \Omega_r = 1$.



4 Summary

The measured cosmological expansion acceleration means that our Universe will expand eternally. The condition of the closed universe eternal expansion restricts the Friedmann integrals ratio by the correlation (8). Written down for the density parameters Ω_m , Ω_v this correlation (13) separate the area in the phase-space (Fig. 2). The measured parameters Ω_m , Ω_v of our Universe are found in this area. The Friedmann integrals of our Universe do not coincide. The phase curves of the eternally expanding closed universes have been plotted. The dependence of the cosmological parameters Ω_m , Ω_v , q , H on the universe curvature radius a has been investigated.

References

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